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Abstract

Based on the introduction of the new adaptive probability for chromosome cross and chromosome mutate, this paper presents a new method for constructing chromosome. A new genetic algorithm for an integrated model of a single finished product VMI (Vendor Managed Inventory) system considering markets and one level BOM (Bill of Materials) is developed. An example is given to illustrate the convergence property and the convergence efficiency of the algorithm. Simulation shows that this method is effective.

Keywords: Vendor managed inventory, Genetic Algorithm, semi-feasible direction

1. Introduction

With the development of information technology, appearance of e-commerce and aggravation global competition, supply chain management is to become brand-new management thought, arouse the attention from entrepreneurs and numerous scholars (Jayashankar M. S. et al, 2003). VMI (Vendor Managed Inventory) is a kind of effective cooperative form of supply chain, can improve the whole competitiveness of supply chain (Jonah and Hui-Ming 2003). A VMI system has been widely adopted by many industries for years. The classical success story for VMI system is found in the partnership between Wal-Mart and Procter & Gamble (P&G). High-tech industries such as Dell, HP and ST Microelectronics also operate efficient supply chains through VMI to reduce inventory levels and costs (Baljko, J.L., 1999).

VMI system was studied in many literatures. Banerjee and Banerjee (1992) consider an EDI-based vendor managed inventory system in which the vendor makes all replenishment decisions for his/her buyers to improve the joint inventory cost. Disney et al (2003) investigates the impact of a VMI strategy upon transportation operations in a supply chain. But most of these works do not consider raw material procurement decisions except for the work by Woo et al (2001). Goyal et al (1995) establish an integrated model and develop a computer program for finding the optimal solution by using an exhaustive search method and consume a large amount of computer resource. Calculating the optimal solution for the model are really a challenges as reported in the literatures. Fortunately GAs (Genetic Algorithms) has been demonstrated successful in providing good solutions to many complex optimization problems and thus received increasing attentions.

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Yugang YU (2003) gives an overall extension to VMI system in which the production of finished products, procurement of multiple raw materials, pricing and advertisement promotion are considered, and introduces an integrated VMI model on a single finished product VMI system in considering markets and one level BOM. The simple genetic algorithm is conducted to solving the model. But our result of measure and test shows that simple genetic algorithm is not as good as self adaptive Genetic Algorithm of this paper whether in the convergent property or in the convergent efficiency. In the paper a new genetic algorithm for solving this model is developed by introducing a new adaptive crossover and mutation probability. The structure of the genetic algorithms is provided and the genetic operators are discussed. In the end an example is given to illustrate the convergence property and the convergence efficiency of the algorithm. Simulation shows that this method is effective.

2. Integrated Model

The VMI system includes vendors and retailers where they can carry on effective cooperation in market pricing and advertisement promotion. This kind of model shows that there is a potential profit in the supply chain and offer powerful foundation for the cooperation and mixing between entrepreneurs in the supply chain. The supply chain system in Yugang YU (2003) consider one vendor and multi-retailers where they can consider the issue standing in the whole supply chain and pursue the maximize profit of the whole VMI system.
The assumption of the model based on VMI is as following:

The vendor who is the administrator in the whole supply chain purchases multiple raw materials to produce only one finished product and let a lot of retailers to sell it. The retailers is scattered in geography and the market is separate. The demand of the market that they face is influenced by the market price and advertising expense of manufacturer and retailers. With the increase of the advertising expense, the demand of market increase, but there is fewer and fewer increasing demand. With the increase of the market price, the demands of every market reduce, but there is fewer and fewer reducing demand. Manufacturer produces products with a regular speed, supplies each retailer’s stock definitely with common order cycle. The retailers can consider the OOS (Out of Stock) because the supply is not in time. The manufacturer undertakes expenses of OOS.

In order to economize space, the initial model and relevant explanation of symbols is to be shown in appendix. The treatment course of the model is to be found in the list of reference (Yugang YU 2003). This model can be divided into two kinds of situations in which can be solved separately.

The first kind of situation is that the manufacturer has excess production capacity ($x=1$). The corresponding mathematic model, notified by VMIIMSP-1, is as follows:

$$\max \quad NP_1 = \sum_{i=1}^{m} D_i(p_i, a_i, A)p_i - \sqrt{2H_1\sum_{i=1}^{m} D_i(p_i, a_i, A)(\sum_{i=1}^{m} S_{bi} + S_p + \sum_{i=1}^{m} S_{\phi_i} + \sum_{j=1}^{l} S_{rj} / n_j)}$$
$$- A - \sum_{i=1}^{m} a_i - \sum_{i=1}^{m} D_i(p_i, a_i, A)(c_m + \phi_i + \sum_{j=1}^{l} M_j c_{rj}) \tag{1}$$

Subject to:

$$\sum_{i=1}^{m} D_i(p_i, a_i, A) < P \tag{2}$$

$$\begin{cases} n_j = 1 & if \quad V_1 + \sum_{k=1,k\neq j}^{l} M_k H_{rk} n_k \leq 0 \\ n_j(n_j-1) < (V_1 + \sum_{k=1,k\neq j}^{l} M_k H_{rk} n_k) \dfrac{S_{rj}}{M_j H_{rj}(T_1 + \sum_{k=1,k\neq j}^{l} \dfrac{S_{rk}}{n_k})} \leq n_j(n_j+1) & otherwise \end{cases} \quad (3)$$

Where:

$$V_1 = [H_p \sum_{i=1}^{m} \frac{D_i(p_i,a_i,A)^2}{P} + \sum_{i=1}^{m} \frac{D_i(p_i,a_i,A) L_{bi} H_{bi}}{L_{bi}+H_{bi}}] / (\sum_{i=1}^{m} D_i(p_i,a_i,A))$$
$$- \sum_{k=1}^{l} [M_k H_{rk} (1 - \sum_{i=1}^{m} D_i(p_i,a_i,A)/P)]$$

$$T_1 = \sum_{i=1}^{m} S_{bi} + S_p + \sum_{i=1}^{m} S_{\phi_i}$$

$$H_1 = [H_p \sum_{i=1}^{m} \frac{D_i(p_i,a_i,A)^2}{P} + \sum_{i=1}^{m} \frac{D_i(p_i,a_i,A) L_{bi} H_{bi}}{L_{bi}+H_{bi}}] / (\sum_{i=1}^{m} D_i(p_i,a_i,A))$$
$$+ \sum_{j=1}^{l} [M_j H_{rj} (n_j - 1 + \sum_{i=1}^{m} D_i(p_i,a_i,A)/P)]$$

Decision variables: n_j is a positive integer； A , p_i and a_i are non-negative numbers, $i = 1,2,\cdots,m$, $j = 1,2,\cdots,l$ 。

Constrain (2) is the existent condition for model VMIIMSP-1; Constrain (3) is needed rule that optimization $(\overline{n}_1^*, \overline{n}_2^*, \cdots, \overline{n}_l^*)$ must meet. In order to prevent it from become one of strong constrains, it can consider properly to relax low bound from $n_j(n_j-1)$ to $n_j(n_j-2)$ while the upper bound is relax from $n_j(n_j-1)$ to $n_j(n_j-2)$.

The optimal out of stock rate in the model is:

$$\overline{b}_i^* = \frac{H_{bi}}{H_{bi} + L_{bi}} \qquad i = 1,2,\cdots,m \qquad (4)$$

The optimal common replenishment cycle in the model is:

$$\overline{C}_1^* = \sqrt{2(\sum_{i=1}^{m} S_{bi} + S_p + \sum_{i=1}^{m} S_{\phi_i} + \sum_{j=1}^{l} \frac{S_{rj}}{n_j^*}) / (H_1 \sum_{i=1}^{m} D_i(p_i^*, a_i^*, A^*))} \qquad (5)$$

Second situation is that manufacturer has insufficient production capacity ($x = 0$). $\sum_{i=1}^{m} D_i(p_i,a_i,A) = P$ is the equality constraint of the model that is unfavorable to solving the problem for GA. It should be noticed that A in the $D_i(p_i,a_i,A(p,a))$ can be regarded as function of $p = (p_1,p_2,\cdots,p_m)$ and $a = (a_1,a_2,\cdots,a_m)$. A is the unique positive solution that can be solved from equation $\sum_{i=1}^{m} D_i(p_i,a_i,A) = P$ when $p_1,p_2,\cdots,p_m$, $a_1,a_2,\cdots,a_m$ are produced at random by GA. It could be solved by a common procedure and reduce the number of the variable (as A is solved) as well as equality restrain dispelled. So in the second situation the model, notified by VMIIMSP-2, is as follows:

$$\max \quad NP_2 = \sum_{i=1}^{m} D_i(p_i,a_i,A(p,a)) p_i - \sqrt{2H_2 P (\sum_{i=1}^{m} S_{bi} + \sum_{i=1}^{m} S_{\phi_i} + \sum_{j=1}^{l} \frac{S_{rj}}{n_j})}$$
$$- A(p,a) - \sum_{i=1}^{m} a_i - \sum_{i=1}^{m} D_i(p_i,a_i,A(p,a))(c_m + \phi_i + \sum_{j=1}^{l} M_j c_{rj}) \qquad (6)$$

Subject to:

$$\begin{cases} n_j = 1 & if \quad V_2 + \sum_{k=1,k\neq j}^{l} M_k H_{rk} n_k \le 0 \\ n_j(n_j-1) < (V_2 + \sum_{k=1,k\neq j}^{l} M_k H_{rk} n_k) \dfrac{S_{rj}}{M_j H_{rj}(T_2 + \sum_{k=1,k\neq j}^{l} \frac{S_{rk}}{n_k})} \le n_j(n_j+1) & otherwise \end{cases} \quad (7)$$

Where:

$$V_2 = \frac{1}{P}[H_p \sum_{i=1}^{m} \frac{D_i(p_i,a_i,A)^2}{P} + \sum_{i=1}^{m} \frac{D_i(p_i,a_i,A)L_{bi}H_{bi}}{L_{bi}+H_{bi}}]$$

$$T_2 = \sum_{i=1}^{m} S_{bi} + \sum_{i=1}^{m} S_{\phi_i}$$

$$D_i(p_i,a_i,A) = K_i \frac{a_i^{e_{a_i}} A^{e_{A_i}}}{p_i^{e_{p_i}}} \qquad i = 1,2,\cdots,m$$

$$H_2 = \frac{1}{P}[H_p \sum_{i=1}^{m} \frac{D_i(p_i,a_i,A)^2}{P} + \sum_{i=1}^{m} \frac{D_i(p_i,a_i,A)L_{bi}H_{bi}}{L_{bi}+H_{bi}}] + \sum_{j=1}^{l}[M_j H_{rj} n_j]$$

Decision variables: n_j is a positive integer; p_i and a_i are non-negative numbers, $i = 1,2,\cdots,m$, $j = 1,2,\cdots,l$ 。

The optimal out of stock rate in the model is:

$$\overline{b_i}^* = \frac{H_{bi}}{H_{bi} + L_{bi}} \qquad i = 1,2,\cdots,m \quad (8)$$

The optimal common replenishment cycle in the model is:

$$\overline{C_2}^* = \sqrt{2(\sum_{i=1}^{m} S_{bi} + \sum_{i=1}^{m} S_{\phi_i} + \sum_{j=1}^{l} \frac{S_{rj}}{n_j})/(H_2 P)} \quad (9)$$

3. Designing of AGA

Genetic algorithms can carry on searching for the overall situation. It adopted colony searching tactics not depend on gradient message of objective function, so there is higher scope of application than the traditional optimization method in solving the complicated engineering problems. GA increasingly demonstrates its superiority. Their uses have been well documented in the literatures, such as that of Goldberg (1989), Michalewicz(1994), Fogel (1994) and CHEN Guo-liang et al(2001) for a wide variety of optimization problems. GA (genetic algorithm) has also been applied to supply chain optimization.

In this section a new Adaptive Genetic Algorithm for solving the integrated model is developed by introducing a new adaptive crossover and mutation probability. The genetic operators are discussed and the structure of the genetic algorithms is provided.

3.1 Selection operator

Roulette wheel selection method adopted in this paper is a direct proportional policy by which the new population can be selected. Its main steps are (Li Shiyong 1998):

a) Compute the fitness for each individual chromosome X_k: $eval(X_k) = f(r,w)$, $k = 1,2,\cdots,pop\_size$;

b) Compute the sum of fitness of all the chromosomes: $F = \sum_{k=1}^{pop\_size} eval(X_k)$;

c) Calculate selection rate for each individual X_k: $p_k = eval(X_k)/F$;

d) Calculate the accumulating probability $q_k = \sum_{j=1}^{k} p_j$, $k = 1,2,\cdots, pop\_size$

pop_size is population's size. The procedure of selection is to turn roulette wheel pop_size times. In every time an individual is selected to the new population. At first it is generated a random number c in [0, 1]. If $c \le q_1$, individual X_1 is selected; Otherwise choose the k-th chromosome X_k if $q_{k-1} < c \le q_k$, $(2 \le k \le pop\_size)$.

3.2 Adaptive crossover operator

In the paper an one-point crossover operator is adopted: Choose an exchange location at random on the parents' coding string, exchange the substrings between two parents from the selected location to the end, and then produce two new individuals.

The fetching value range of commonly used optimal crossover probability is from 0.5 to 0.95. The relation between the adaptive crossover probability and relatively genetic generations is a hyperbola. The formula is:

$$p_c^t = \begin{cases} \dfrac{p_{c,\max}}{1 + t / t_{\max}} & p_c^t > p_{c,\min} \\ p_{c,\min} & p_c^t \le p_{c,\min} \end{cases} \tag{10}$$

Among them: p_c^t is crossover rate of the t-th generation. $p_{c,\max}$ is the maximal cross-over rate; $p_{c,\min}$ is the minimal crossover rate; t is the generation. $t_{\max}$ is the maximal generation. The relation between the designed crossover probability and relatively evolving generations are as Fig. 1 shows:

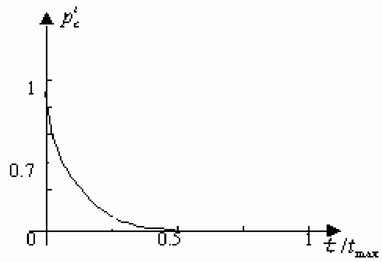

Fig. 1: Relation between crossover probability and relatively evolving generations

From the Fig.1, it shows that the crossover rate of this paper can get biggish probability to generate a new population on the early stage of evolution and improve the diversity of the populations; On the later stage of evolution, the crossover probability of this paper get a lesser probability and its value is steady at 0.5. It would be helpful to keep the elitist individuals. In this way, it will improve the efficiency of searching and lead to a higher convergent speed.

3.3 Adaptive mutation operator

If the mutation p_m is small in the later stage of searching in GA, convergent speed will be very slow. Oppositely if p_m is too big, there are mutating in several positions of gene series, and it is hard to converge.

Assuming that X_0 mutate and change into X_1, the more probability that X_1 is superior than X_0, the more advantages in evolution there are.

Theorem [Zhang Liangjie 1996]: When X_0 mutate to create X_1 and enter the i-bit improved sub-space, the probability $P(eval(X_1) \geq eval(X_0))$ of improving the subspace reaches the maximum when the mutation probability is $p_m = i/N$ where N is the length of the gene series.

According to this theorem, the adaptive mutation rate in the paper is inverse proportion to the relatively Euclid distance among the parents' strings and decrease with respect to the relative genetic generation. The formula is:

$$p_m^t = \begin{cases} p_{m,\max} \cdot \exp(-\sigma t / t_{\max}) \cdot (1 - R / R_{\max}) & p_m^t > p_{m,\min} \\ p_{m,\min} & p_m^t \leq p_{m,\min} \end{cases} \quad (11)$$

p_m^t is mutation rate of the t-th generation. R is Euclid distance among the parents' strings and $R_{\max}$ is the max Euclid distance among the parents' strings. $t_{\max}$ is the maximum number of generation. $p_{m,\max}$ is the maximum mutating probability and $p_{m,\min}$ is the minimum mutating probability. σ is a constant.

3.4 Design of GA

(1) Given the input and control parameters, the genetic parameters: population size is pop_size; The parameter $t_{\max}$ is the maximum generation; $p_{c,\max}$ is the maximum crossover probability; $p_{c,\min}$ is the minimum; $p_{m,\max}$ is the maximum mutating probability and $p_{m,\min}$ is the minimum mutating probability; σ is a constant.

(2) Initial population: Randomly generate an initial population $E^n = (X_1, X_2, \cdots, X_n)$.

(3) Evaluation: Using formulas (1), (2), (3) in the model VMIIMSP-1 and formulas (6), (7) in the model VMIIMSP-2 to evaluate population and computing selection probability $p_k = eval(X_k / \sum_{j=1}^{pop\_size} eval(X_j)$.

(4) Selection: Using evaluation function to select individual into new population through roulette-wheel selection.

(5) Crossover: Adaptive crossover with the probability calculated by formula (10).

(6) Mutation: Adaptive mutation with the probability calculated by formula (11).

(7) Stop condition check: If request of counting precision and the max step satisfied, stop; otherwise go to (3).

4. Simulative Examination

The assumptions of the model have been given out in section 2. The elitist individual is copied to next generation to ensure the convergence for the AGA and SGA (CHEN Guo-liang et al, 2001).

In order to compare conveniently, the example in reference (Yugang YU, 2003) is adopted to carry on simulative examination. The corresponding input parameters are given in table 1 which is the case of $m = 3, l = 2$. The monetary unit is U.S. dollar and the unit time is "year".

Table 1: The VMIIMSP Model in Simulation

Parameter	c_m	c_{rj}	e_{A_i}	H_{bi}	H_{rj}	l	m	P	S_p	S_{ϕ_i}
Value	20	20	0.39	12	2	2	3	50000	200	20
Parameter	e_{a_i}	e_{p_i}	H_p	K_i	L_{b_i}	M_j	S_{bi}	S_{rj}	ϕ_i	—
Value	0.43	1.3	4	350	500	1.1	80	500	10	—

The SGA program using Java Builder 8.0 is developed in Yugang YU (2003). The parameters and the results are given out in table 2 and table 3 respectively.

Table 2: Test Parameters and the Test Analysis of SGA in Yugang YU (2003)

Maximal generation	$t_{\max} = 1000$	Mutation rate	$p_m = 0.3$
Population size	$pop\_size = 100$	Convergent generation	1000
Crossover rate	$P_c = 0.4$	CPU time（S）	5.31

Table 3: Results of the Integrated Model in Yugang YU (2003)

a_i	b_i	n_j	p_i	C	A	x	NP
9788378.10	0.0234	3	1775.58	0.031	26633373.64	0	29039004.32

In order to compare in same conditions, both the SGA and AGA programs are designed and the toolbox in application software Matlab is adopted to write the programs. Simulative examinations is carried on the computer of P4，256M EMS memory.

The parameters and the results of SGA in our worked are given out in table 4 and table 5 respectively.

Table 4: Test Parameters and the Test Analysis of SGA

Maximal generation	$t_{\max} = 1000$	Mutation rate	$p_m = 0.05$
Population size	$pop\_size = 100$	Convergent generation	1000
Crossover rate	$P_c = 0.6$	Average CPU time（S）	492

Table 5: Results of the Integrated Model Using SGA

a_i	b_i	n_j	p_i	C	A	x	NP
9870311.88	0.0234	3	1783.77	0.0225	26798114.65	0	29038944.92

The optimal value of the object function using SGA in our experiment is 29038944.92 and is less accuracy than the optimal value that being received in Yugang YU (2003). It is noticed that the parameters between table 2 and table 4 are deferent because the suitable optimal values using parameters from table 2 cannot be received in our tests.

The parameters and the results of AGA in our worked are given out in table 6 and table 7 respectively.

Table 6: Test Parameters and the Test Analysis of AGA

Maximal generation	$t_{\max} = 1000$	Maximum mutation rate	$p_{m,\max} = 0.45$
Population size	$pop\_size = 100$	Minimum mutation rate	$p_{m,\min} = 0.001$
Maximal crossover rate	$p_{c,\max} = 0.95$	Best convergent generation	328
Minimal crossover rate	$p_{c,\min} = 0.5$	Average CPU time（S）	299

Table 7: Results of the Integrated Model Using AGA

a_i	b_i	n_j	p_i	C	A	x	NP
9784246.68	0.0234	3	1775.17	0.023	26625567.22	0	29039527.09

Constant $\sigma = 10$. The time of CPU is equal to the time that is needed in generating the optimal value. The best convergent generation and the time of CPU are the average values that are received by several experimentations.

When the adaptive GA searches in the space, the fitness increases with the training time increasing and converges quickly. The optimal value of the integrated model is the bigger optimal value between VMIIMSP-1 and VMIIMSP-2. The optimal value （29039527.09） received by AGA in this paper is more accuracy than both the optimal value(29038944.92) received by SGA and the optimal value(29039004.32) in Yugang YU (2003).

The simulative result indicates that the optimal values are different when the crossover probability and mutating probability are varied in SGA. The better feasible solution is obtained using better probabilities that are gained by carrying on many experiments. On the contrary, all the relatively parameters in adaptive GA are given out at one time only and the chromosome space structured in this paper can get a nice feasible solution by using an adaptive crossover operator and a mutating operator as genetic operators. This AGA has higher performance than Simple GA in receiving the feasible solution and consumes less time. The adaptive GA is the ideal algorithm in solving this question.

5. Conclusion

This paper designs an adaptive GA for an integrated model of a single finished product VMI (Vendor Managed Inventory) system considering markets and one level BOM (Bill of Materials). And the structure and genetic operators of the AGA are discussed. In evolutionary, adaptive crossover rate and mutation rate, which consider the relationship between probability change and the number of generation, are introduced. With the number of generation increase, probability is adaptive changes accordingly that can guarantee the diversity of the population and made the AGA have a higher convergent speed. The simulative experiment has proved that this adaptive GA is effective.

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Appendix:

1. Integrated Model:

Model VMIIMSP：
Objective function:

$$\max \quad NP=\sum_{i=1}^{m} D_i(p_i,a_i,A)p_i-\frac{1}{C}[\sum_{i=1}^{m} S_{bi}+xS_p+\sum_{i=1}^{m} S_{\phi_i}+\sum_{j=1}^{l} S_{rj}/n_j]$$
$$-\frac{C}{2}[\sum_{i=1}^{m} D_i(p_i,a_i,A)(1-b_i)^2 H_{bi}+\sum_{i=1}^{m} D_i(p_i,a_i,A)b_i^2 L_{bi}+H_p\sum_{i=1}^{m} D_i(p_i,a_i,A)^2/P$$
$$+\sum_{j=1}^{l} M_j H_{rj}\sum_{i=1}^{m} D_i(p_i,a_i,A)(n_j-1+\sum_{i=1}^{m} D_i(p_i,a_i,A)/P)]-A-\sum_{i=1}^{m} a_i$$
$$-\sum_{i=1}^{m} D_i(p_i,a_i,A)(c_m+\phi_i+\sum_{j=1}^{l} M_j c_{rj}) \qquad (1)$$

Constraints:

$$\sum_{i=1}^{m} D_i(p_i,a_i,A) \le P \qquad (2)$$

$$P-\sum_{i=1}^{m} D_i(p_i,a_i,A) \le xM \qquad (3)$$

$$0 \le b_i \le 1 \quad i=1,2,\cdots,m \qquad (4)$$

$$D_i(p_i,a_i,A)=K_i\frac{a_i^{e_{a_i}} A^{e_{A_i}}}{p_i^{e_{p_i}}} \qquad i=1,2,\cdots,m \qquad (5)$$

Decision Variables:

n_j is a positive integer; A, p_i and a_i are nonnegative numbers, $i=1,2,\cdots,m$, $j=1,2,\cdots,l$; x is a 0-1 variable. M is an infinite positive number. Constraint (3) is show whether there is a surplus productive capacity. If there is a surplus productive, the corresponding condition is $\sum_{i=1}^{m} D_i(p_i,a_i,A)<P$ and $x=1$, and startup cost S_p is considered in net profit. Otherwise, $x=0$ and $\sum_{i=1}^{m} D_i(p_i,a_i,A)=P$, startup cost S_p is not considered in net profit because equipments are working continually. Constraint (4) shows that OOS rate of the retailers are between complete and sufficient. Constraint (5) is demanding function that can be canceled by substituting into the objective function.

This model can be divided into two kinds of conditions as the paper has been shown.

2. Notations:

Fixing Parameters and Function:

m	Number of retailers considered
$i=1,2,\cdots,m$	Index of retailers
l	Number of raw materials considered
$j=1,2,\cdots,l$	Index of raw materials
$D_i(p_i,a_i,A)$	Demand for retailer i per unit time
c_m	Production cost per unit finished products

c_{rj}	The price of raw materials j
e_{a_i}	Demanding elasticity of cost in advertisement of retailer i
e_{A_i}	Demanding elasticity of cost in advertisement of vendor in markets of retailer i
e_{p_i}	Demanding elasticity of price of retailer i
H_{bi}	Holding cost of the finished products for retailer i
H_p	Holding cost of the finished products for the vendor per time
H_{rj}	Holding cost of the raw materials
K_i	Constraint in demanding function of retailer i
L_{b_i}	Cost of OOS of retailer i per unit per time
M_j	The needing of raw materials j in producing per unit, it shows the relationship between products BOM and raw materials
NP	Total profits in VMI per time
P	The producing rate
S_{bi}	Fixed cost for vendor manages inventory of retailer i ($/time)
S_p	Startup cost for vendor per time
S_{rj}	Ordering cost per raw material j for vendor
S_{ϕ_i}	Fixed cost for the vendor replenishing production to retailer i
t_i	Time for the vendor supplying the demanding of retailer i in the common ordering cycle
ϕ_i	Transport cost from vendor to retailer per unit

Decision Variables:

A	Advertising cost of vendor
a_i	Advertising cost of retailer i
b_i	Proportion of OOS of retailer i per supply cycle
C	Common ordering cycle for the vendor and all the retailers
n_j	Procurement cycle of the raw material j which is an integer number. It is a multiple of the common ordering cycle.
p_i	Market price of retailer i
x	a 0-1 variable